Hold onto Hwk: we will turn in today's homework on Wednesday,

along with Wednesday's hwk:

Monday	Tuesday	Wednesday	Thursday	Friday
Jan 23 1.7abd, 3.60bcef, 6.7bckl	Jan 24	Jan 25 6.9abd, 6.10abc	Jan 26	Jan 27 6.11cfghikmn

Note changes

Wed: pile 1 pile 2

New technique: integration by parts
("product rule backwards")

Product Rule: $u = u(x)$
$v = v(x)$

$$(uv)' = u'v + uv'$$

Integrating both sides,

$$\int (uv)'\,dx = \int u'v\,dx + \int uv'\,dx$$

du dv

$\Rightarrow \quad uv = \int v\,du + \int u\,dv$

solve for this

$$\boxed{\int u\,dv = uv - \int v\,du}$$

integration by parts formula.

sometimes we have to use it more than once.

Key method. $\underbrace{\int \_\_\_\_\_\_ \,dx}_{\text{find } u \text{ and } dv \text{ in here.}} = ?$

find u and dv in here.

Big question: what do you let $u = ?$

Rules of thumb:

① $\int \underbrace{\overset{(3x^2-2x+1)}{(\text{polynomial})}}_{u} \underbrace{\overset{\sin(5x)}{(\text{transcendental fcn})}\,dx}_{dv}$

② $\int \overset{3x^2-2x+1}{(\_\_\_\_)} \underbrace{\overset{\ln(x),\ (\arctan(x))^2}{(\text{inverse fcn})}}_{u}\,dx$

dv

Examples

① $\int x\cos(5x)\,dx \overset{?}{=}$

parts: let $u = x \Rightarrow du = 1\,dx$

$dv = \cos(5x)dx \xRightarrow[\text{antideriv.}]{} v = \frac{1}{5}\sin(5x)$

$= uv - \int v\,du$

$= x\frac{1}{5}\sin(5x) - \int \frac{1}{5}\sin(5x)\,dx$

$= \frac{1}{5}x\sin(5x) - \left(-\frac{1}{25}\cos(5x)\right) + C$

$= \boxed{\underbrace{\frac{1}{5}x\sin(5x) + \frac{1}{25}\cos(5x) + C}_{\text{Bubba}}}$

Check $(\text{Bubba})' = \cancel{\frac{1}{5}\sin(5x)} + \frac{1}{5}x\cos(5x)\cdot 5 + \cancel{-\frac{1}{25}\sin(5x)5}$

$= x\cos(5x)$

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② $\int \underbrace{(3x^2-2x)}_{u}\overbrace{\cos(5x)dx}^{dv} = ?$

parts: let $u = 3x^2-2x$ $du = (6x-2)dx$

$dv = \cos(5x)dx \Rightarrow v = \frac{1}{5}\sin(5x)$

$= uv - \int v\,du$

$= (3x^2 - 2x)\left(\frac{1}{5}\sin(5x)\right) - \int \frac{1}{5}\sin(5x)(6x-2)\,dx$

$= \frac{1}{5}(3x^2-2x)\sin(5x) - \frac{1}{5}\int (6x-2)\sin(5x)\,dx$

parts again

$u = 6x-2 \quad du = 6\,dx$

$dv = \sin(5x)\,dx \quad v = -\frac{1}{5}\cos(5x)$

$= \frac{1}{5}(3x^2-2x)\sin(5x) - \frac{1}{5}\left[-\frac{1}{5}(6x-2)\cos(5x) + \frac{6}{5}\underbrace{\int \cos(5x)\,dx}_{\frac{1}{5}\sin(5x)}\right]$

$= \frac{1}{5}(3x^2-2x)\sin(5x) + \frac{1}{25}(6x-2)\cos(5x) - \frac{6}{125}\sin(5x) + C$

Checking your work:

8:23 PM Mon Jan 23

Ken Richardson's Home | Calculus II | Sage Cell Server | New Tab

sagecell.sagemath.org

Type some Sage code below and press Evaluate.

```
f(x)=x*cos(5*x)
show(integral(f(x),x))
```

Evaluate

$$\frac{1}{5}\,x\sin\left(5\,x\right) + \frac{1}{25}\,\cos\left(5\,x\right)$$

Board work during blackout:

③ $\int (\ln(x))^2 \, dx$

$u = (\ln(x))^2 \qquad du = 2\ln(x) \cdot \frac{1}{x} \, dx$

$dv = dx \qquad v = x$

$$= uv - \int v \, du$$

$$= (\ln(x))^2 x - \int x \, 2\ln(x) \frac{1}{x} \, dx = x(\ln(x))^2 - 2\Big(\underbrace{\int \ln(x) \, dx}_{\text{parts}}\Big)$$

parts $u = \ln(x) \quad du = \frac{1}{x} dx$

$dv = dx \quad v = x$

$$= x(\ln(x))^2 - 2\Big[\ln(x) \cdot x - \underbrace{\int x \frac{1}{x} dx}_{\int 1 dx \;=\; x}\Big] = \boxed{x(\ln(x))^2 - 2x\ln(x) + 2x + C}$$

④ $\int e^x \cos(x) \, dx = e^x \sin(x) - \int \sin(x) e^x \, dx = e^x \sin(x) - \left[e^x(-\cos(x)) - \int (-\cos(x)) e^x \, dx\right]$

$\boxed{\begin{array}{l} \text{Parts: let } u = e^x \Rightarrow du = e^x dx \\ dv = \cos(x) dx \Rightarrow v = \sin(x) \end{array}}$

parts $u = e^x \Rightarrow du = e^x dx$

$dv = \sin(x) dx \Rightarrow v = -\cos(x)$

$$\int e^x \cos(x) \, dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x) \, dx$$

$$\Rightarrow 2\int e^x \cos(x) \, dx = e^x \sin(x) + e^x \cos(x)$$

$$\boxed{\int e^x \cos(x) dx = \tfrac{1}{2} e^x \sin(x) + \tfrac{1}{2} e^x \cos(x) + C}$$